Multi-channel Real-time Computation of ADEV and TDEV

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INTRODUCTION

Allan deviation ADEV and time deviation TDEV are commonly used for describing the quality of synchronization signal in the telecommunication network [1, 2, 3]. These parameters allow the variations of time interval provided by the synchronization signal to be assessed and the type of phase noise affecting the signal to be recognized. The estimates of the parameters are computed for a series of observation intervals using the sequence of time error samples previously measured at some network interface. The application of real-time computation of the parameters performed during time error measurement process allows to reduce the evaluation time.

One may occur, that more than one timing signal has to be analyzed in the same time. Such situation can be considered in the node of the telecommunication network, where the signals arrive from several directions, or when we want to perform the three hat corner procedure. In such a case, a multi-channel time error measuring system enabling simultaneously measurement of several timing signals is necessary. Additionally, computation algorithms enabling real-time assessment of ADEV or TDEV for many data series could be very helpful.

In this paper the methods of multi-channel real-time computation of ADEV and TDEV are proposed. These methods are developed for a multi-channel measuring system, where the time error counter and the computer controlling the measurement are two separate units. In order to calculate the parameters simultaneously for several channels and several observation intervals in the real time, all necessary operations should be performed in the time period between two sampling instants, i.e. during the sampling interval τ_0. Multi-channel real-time computation requires the same rearrangement of estimators' formulae, as for single-channel real-time computation [6, 7]. However, multi-channel computation requires a special organization of the process because of multiple data streams received from several time error meters (measurement channels).

In the paper the results of experimental tests of the methods proposed for different conditions are presented. The calculations were performed for the time error samples taken with sampling interval τ_0=1/30 s, which is often used in the telecommunication applications. Different numbers, lengths, and ranges of the observation intervals simultaneously analyzed were considered. The results of calculation performed for data obtained from up to 4 channels using different computers are presented and compared. Two data organization methods – with separate data structures (one for each channel) and joint data structure – were considered and compared.

ADEV AND TDEV ESTIMATORS

Allan deviation and time deviation are computed based on the averaging of second differences of the phase process $x(t)$ of the analyzed timing signal. We can assume for the telecommunication applications, in the case of negligible influence of frequency drift, that ADEV and TDEV are estimated based on the time error function measured between the analyzed timing signal and the reference one [4].

The formulae for the estimators of Allan deviation ADEV and time deviation TDEV take the form:

$$A\hat{D}EV(\tau)=\sqrt{\frac{1}{2n^2\tau_0^2(N-2n)}\sum_{i=1}^{N-2n}\left(x_{i+2n}-2x_{i+n}+x_i\right)^2} \tag{1}$$

$$T\hat{D}EV(\tau)=\sqrt{\frac{1}{6n^2(N-3n+1)}\sum_{j=1}^{N-3n+1}\left[\sum_{i=j}^{j+n-1}\left(x_{i+2n}-2x_{i+n}+x_i\right)\right]^2} \tag{2}$$

where $\{x_i\}$ is a sequence of N samples of time error function $x(t)$ taken with interval τ_0; $\tau=n\tau_0$ is an observation interval. For TDEV computation the estimator formula (2) can be changed in order to simplify the calculation of sum [4, 5] and takes the form:

$$T\hat{D}EV(n\tau_0)=\sqrt{\frac{1}{6}\cdot\frac{1}{N-3n+1}\cdot\frac{1}{n^2}\sum_{j=1}^{N-3n+1}S_j^2(n)} \tag{3}$$

where

$$S_j(n)=S_{j-1}(n)-x_{j-1}+3x_{j+n-1}-3x_{j+2n-1}+x_{j+3n-1} \tag{4}$$

$$S_1(n)=\sum_{i=1}^{n}\left(x_{i+2n}-2x_{i+n}+x_i\right) \tag{5}$$

When computing in the real time, we do not have access to the time error samples indexed by $i+n$ or $i+2n$ for the current time instant described by index i, because these samples have not been measured yet. We have access to the sample currently measured (for the current sampling instant i) and the samples measured earlier (with indexes smaller than i) and stored in the equipment's memory. When computing off-line, after the measurement is finished, we have access to the whole data sequence for each measurement channel. There is no difference, if we first compute the parameter's values for each observation interval for one selected data sequence (one channel) or we start compute for one selected observation interval for each data sequence. When computing in the real time, for current sampling instant i we have to perform all operations for each channel and for each observation interval within this sampling interval. The scheme of indexing the samples for real-time computation is presented in Fig. 1. Therefore, the indexes in formulae for ADEV and TDEV estimators given by (2-5) should be changed in the case of real-time calculation.

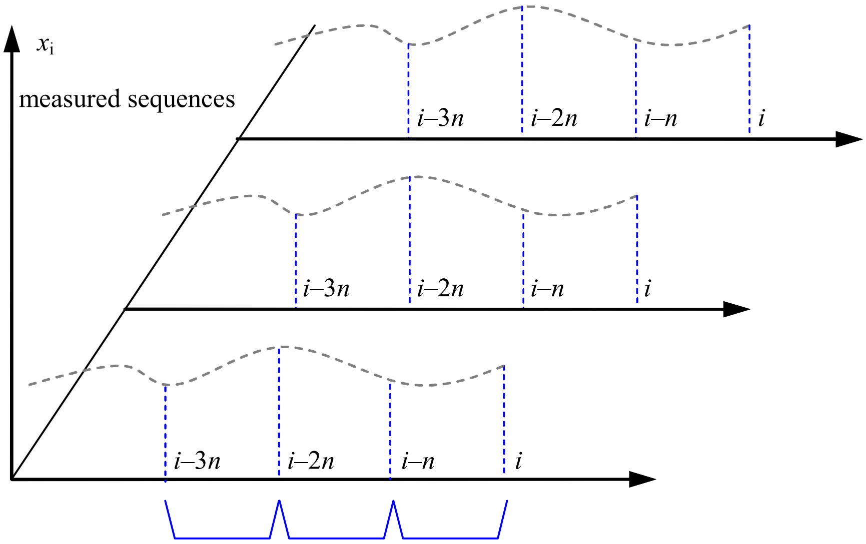

Fig. 1. Samples' indexing for real-time computation for multiple channels

The rearrangement of indexes for the estimators of ADEV and TDEV was performed in [6]. As a result we have obtained the ADEV estimator's formula for a current instant i and selected measurement channel m in the form depending of the sum of squares of second differences computed for the previous sampling instant $i-1$

$${}_mA\hat{D}EV_i(n\tau_0)=\sqrt{\frac{1}{2n^2\tau_0^2(i-2n)}\left({}_mA_{i-1}(n)+\left({}_mx_i-2{}_mx_{i-n}+{}_mx_{i-2n}\right)^2\right)} \tag{6}$$

were ${}_mA_i(n)$ is the sum of squares of second differences of time error samples

$${}_mA_i(n)=\sum_{j=2n+1}^{i}\left({}_mx_j-2{}_mx_{j-n}+{}_mx_{j-2n}\right)^2,\quad i>2n \tag{7}$$

and ${}_mx_i$ denotes the time error sample measured at the sampling instant i in the channel m.

The conversion of the time deviation estimator given in the simplified form (4-6) brought the formula dependent on the overall sum of squares and internal sum computed for the instant $i-1$ and four time error samples

$$ {}_m T\hat{D}EV_i(n\tau_0) = \sqrt{\frac{1}{6}\cdot\frac{1}{i-3n+1}\cdot\frac{1}{n^2}\left[{}_m S_{ov,i-1}(n) + \left({}_m S_{i-1}(n) - {}_m x_{i-3n} + 3{}_m x_{i-2n} - 3{}_m x_{i-n} + {}_m x_i\right)^2\right]} \quad (8) $$

where ${}_m S_{\mathrm{ov},i}(n)$ is the overall sum updated for each sample i, given in the form:

$$ {}_m S_{ov,i}(n) = {}_m S_{ov,i-1}(n) + {}_m S_i^2(n) \quad (9) $$

where

$$ {}_m S_i(n) = {}_m S_{i-1}(n) - {}_m x_{i-3n} + 3{}_m x_{i+2n} - 3{}_m x_{i+n} + {}_m x_i, \quad i > 3n \quad (10) $$

and

$$ {}_m S_{3n}(n) = \sum_{j=2n+1}^{3n}\left({}_m x_j - 2{}_m x_{j-n} + {}_m x_{j-2n}\right), \quad j > 2n \quad (11) $$

The sum ${}_m S_{3n}(n)$ is the first element of the overall sum ${}_m S_{\mathrm{ov},i}(n)$ given by (9). As a result of the conversion of the parameters' formulae, in order to compute ADEV and TDEV for a selected data sequence m (m-th measurement channel), current sampling instant i and given observation interval $\tau=n\tau_0$, we need the values of appropriate sum ${}_m A_{i-1}(n)$, ${}_m S_{\mathrm{ov},i-1}(n)$, and ${}_m S_{i-1}(n)$, currently measured sample ${}_m x_i$ and the samples ${}_m x_{i-n}$, ${}_m x_{i-2n}$, and ${}_m x_{i-3n}$ previously measured and stored in memory.

PROCEDURE OF REAL-TIME MULTI-CHANNEL COMPUTATION

The formulae of the parameters given in the forms presented in the previous section allow us to perform the computation in the real time, during the time error measurement process. The calculation can be performed jointly for both parameters considered, as well as for each parameter separately. A general procedure for the real-time computation of ADEV or/and TDEV [7] is as follows:

1. Measure a new time error sample for each measurement channel and store it in a data file.
2. Compute the appropriated differences for a given n (observation interval $\tau=n\tau_0$) using the current sample, and the samples measured n, $2n$ or $3n$ sampling intervals earlier.
3. Update the appropriated sums.
4. Compute current averages and their square roots.
5. Execute Steps 2-4 for successively greater observation intervals (greater n) for each channel.
6. Return to Step 1 (measure new samples).
7. When the measurement is finished, the values of the parameter's estimate for the observation intervals considered are known.

The computations of ADEV and TDEV start when the sample no. $2n+1$ for each channel has been measured. The first value of ADEV estimate can be computed at this instant. However, for the TDEV the computation of the internal sum $S_{\mathrm{i}}(n)$ only just starts. The first values of TDEV estimate can be computed after the sample no. $3n+1$ has been measured.

We can consider two ways of data organization for multi-channel time error measurement. The schemes of data collecting for both methods are presented in Fig. 2 and 3. In the first method (Fig. 2), the time error samples are collected from each time error meter, sent to the computer and stored separately, in the different data structures. In the second method (Fig. 3), the time error samples are collected from time error meters, aggregated in one data stream, sent jointly to the computer and then stored in one data structure. Each method implies different procedure of parameters' computation performed within one sampling interval. Separate data structures require execution of data reading procedure for a given n separately for each channel (each separate data structure). The object of computation for separate data structures are M scalars (one for each measurement channel). The object of computation for joint data structures is M-dimensional vector (one for each measurement channel). Detailed procedure for the separate data structures will be as follows:

1. Read TE samples from the TE meters and store them in separately data files.
2. Read TE samples measured n, $2n$ or $3n$ sampling intervals earlier from 1. data file.
3. Compute sums according to (7) and (9-11) for 1. channel.
4. Compute current averages and their square roots according to (6) and (8) for 1. channel.
5. Execute Steps 2-4 for successively larger observation intervals.
6. Execute Steps 2-5 for next measurement channels.

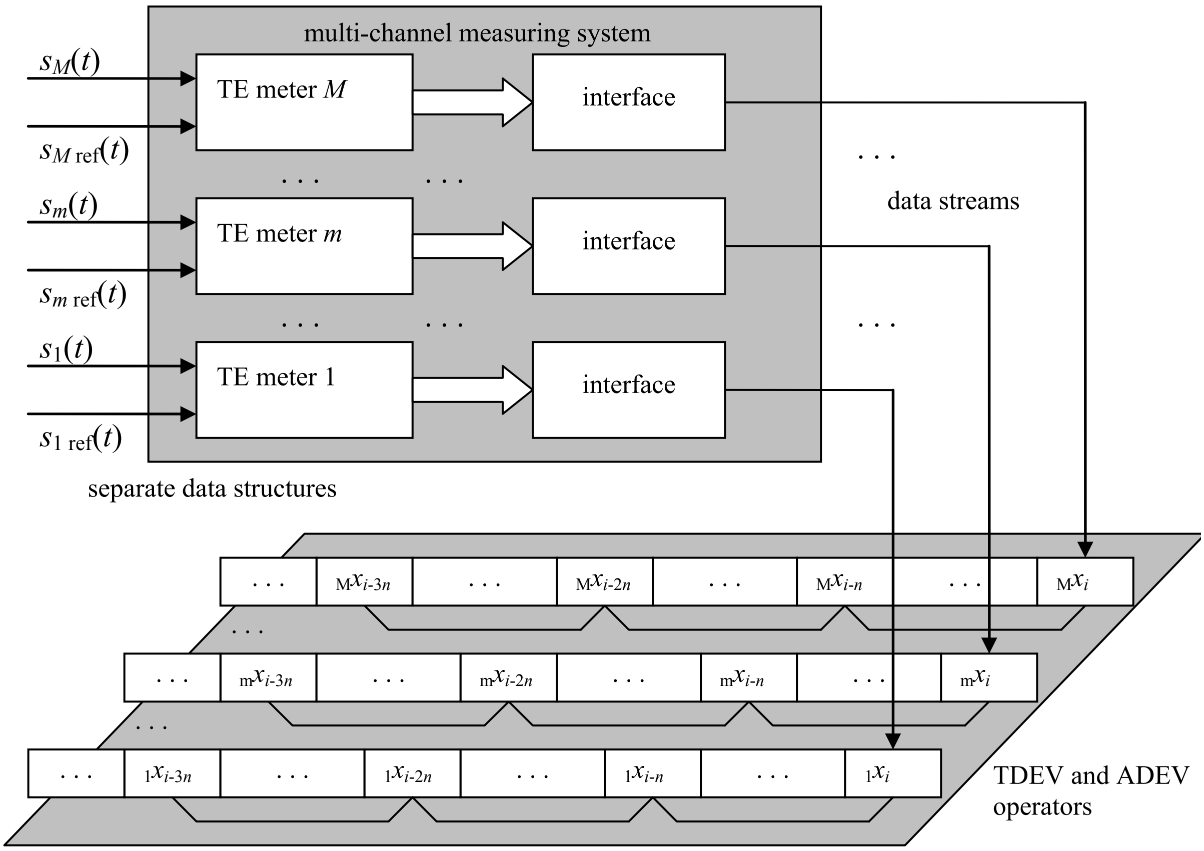

Fig. 2. Multi-channel time error measurement with separate data structures

Application of joint data structure requires single execution of data reading procedure for a given *n*. Detailed procedure for the joint data structure will be as follows:

1. Read TE samples from the TE meters and store them in joint data file.
2. Read TE samples measured *n*, 2*n* or 3*n* sampling intervals earlier for each channel from the data file.
3. Compute sums according to (7) and (9-11) successively for each channel.
4. Compute current averages and their square roots according to (6) and (8) successively for each channel.
5. Execute Steps 2-4 for successive greater observation intervals.

Application of joint data structure makes the procedure simplest than for the separate data structures. Therefore one can expect better performance of the real-time parameters' computation.

COMPUTATION EXPERIMENT

The method of multi-channel real-time computation of time deviation jointly with Allan deviation was tested in the experiment. The experiment was realized similarly as the tests of real-time ADEV and TDEV computation methods presented in [6, 7]. The calculations were performed off-line with the imitation of on-line work. The data sequences created by the multi-channel data stream obtained from the set of time error meters contain time error samples taken with the sampling interval τ_0=1/30 s during the time of 4000 s, representing combination of white phase noise and white frequency noise.

The calculations were performed for variable numbers of observation intervals, arranged in the logarithmic scale in a range between 0.1 s and 1000 s. The starting (smallest) observation interval was τ_{min}=0.1 s (*n*=3) and the longest observation interval was τ_{max}=1000 s (*n*=30000). The calculations were performed for 5, 10, and 20 observation intervals per decade. Both types of data structure, described in the previous section, were considered. The parameters' computations were performed for 1, 2, 3, and 4 measurement channels.

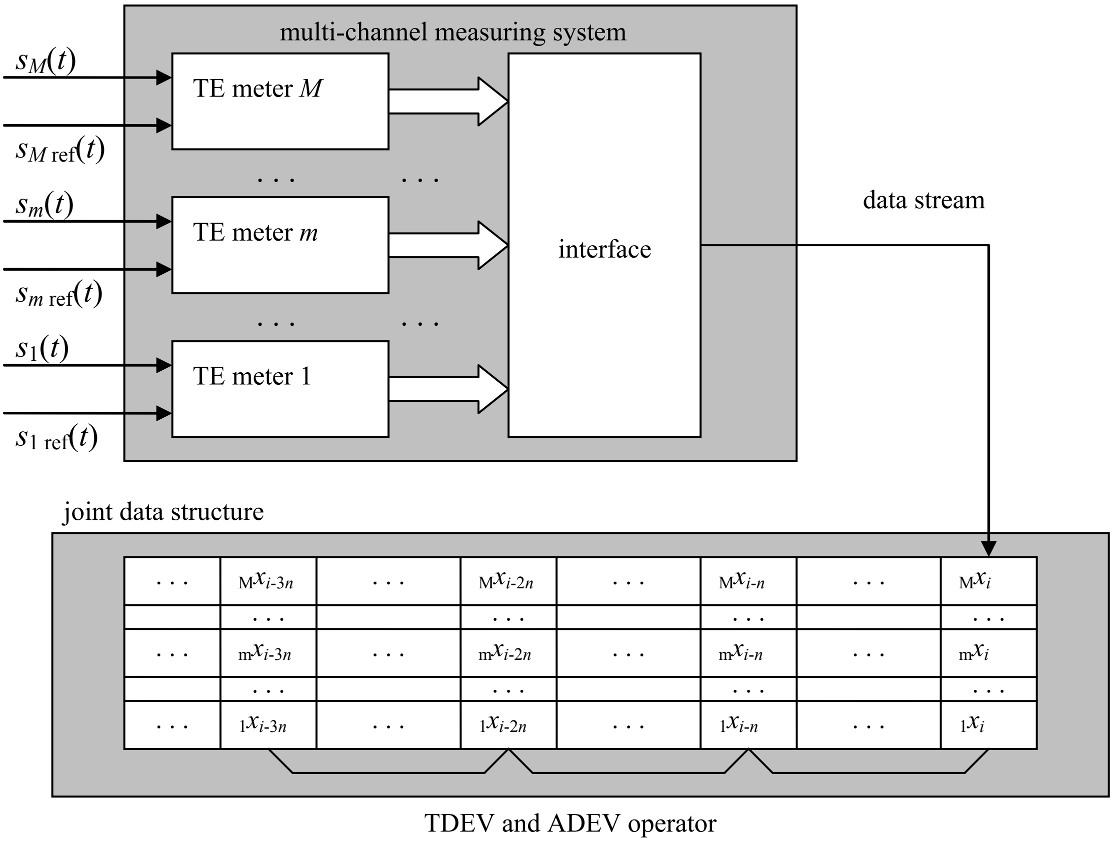

Fig. 3. Multi-channel time error measurement with joint data structure

Two personal computers with Intel Pentium IV 3.0 GHz and Intel Core 2 Quad 3.0 GHz microprocessors were used in the experimental tests. The maximum time *t-max* used for calculation within one sampling interval was the observed quantity. It was assumed that this time cannot exceed the length of the considered sampling interval τ_0=1/30 s = 0.0333… s = 33.3… ms. Calculations were performed for joint computation of TDEV and ADEV. The time of computation for both computers is presented in Table 1 and Table 2. The results of TDEV and ADEV computed for selected instants of the measurement process are presented in Fig. 4 – 6.

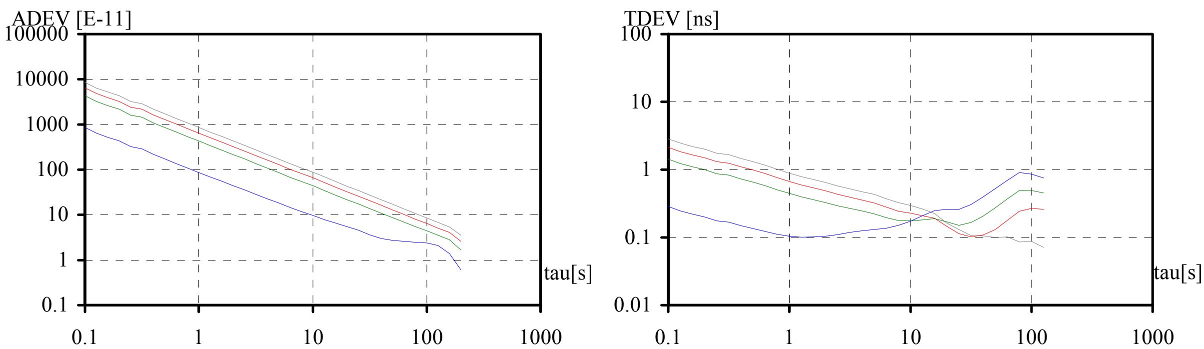

Fig. 4. ADEV and TDEV computed for the measurement instant *i*=12 000 samples (400 s)

The time results obtained for both computers (Table 1 and 2) have proved the expectation, that the application of joint data structure for time error samples obtained from several measurement channels is more time effective than application of separate data structures, if the real-time assessment of ADEV and TDEV is performed. The maximum time used for computation observed for separate data structures increases highly (approximately proportionally to the number of channels) with the number of channels simultaneously considered. Increment of number of channels for joint data structure increases the maximum time insignificantly. The influence of the most critical issue – time to access to previously saved samples obtained from multiple channels – is minimized. However, the time results were satisfactory for all cases considered. The maximum time of operations performed for one sampling interval does not exceed the length of considered sampling interval 1/30 s. The application of computer with more modern microprocessor makes the computation time to be shorter significantly.

Table 1. Time of computation using computer with Pentium IV 3.0 GHz

Number of channels	Separate data structures			Joint data structure		
	Number of intervals per decade			Number of intervals per decade		
	5	10	20	5	10	20
	t-max [ms]	t-max [ms]	t-max [ms]	t-max [ms]	t-max [ms]	t-max [ms]
1	0.64	1.20	2.45	0.64	1.20	2.45
2	1.21	2.35	4.65	0.70	1.37	2.71
3	1.80	3.50	6.93	0.77	1.50	2.97
4	2.39	4.66	9.22	0.84	1.63	3.21

Table 2. Time of computation using computer with Intel Core 2 Quad 3.0 GHz

Number of channels	Separate data structures			Joint data structure		
	Number of intervals per decade			Number of intervals per decade		
	5	10	20	5	10	20
	t-max [ms]	t-max [ms]	t-max [ms]	t-max [ms]	t-max [ms]	t-max [ms]
1	0.23	0.46	0.83	0.23	0.46	0.83
2	0.44	0.88	1.58	0.26	0.50	0.92
3	0.63	1.21	2.38	0.28	0.54	1.01
4	0.83	1.60	3.16	0.30	0.58	1.09

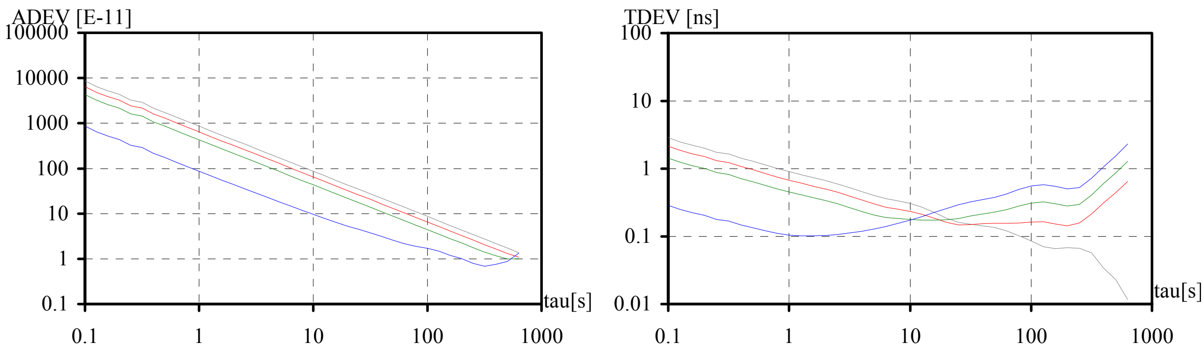

Fig. 5. ADEV and TDEV computed for the measurement instant *i*=60 000 samples (2000 s)

The computation complexity of the parameters considered does not depend on the length of observation interval; the number of observation intervals and the number of channels analyzed simultaneously are the limiting factors. The time results obtained in the experiment show that there is some reserve of computational power of the tested equipment. One can expect good results (maximum time not exceeding the sampling interval) for wider range and greater number of observation intervals, as well as for greater number of measurement channels than the quantities considered in the experiment.

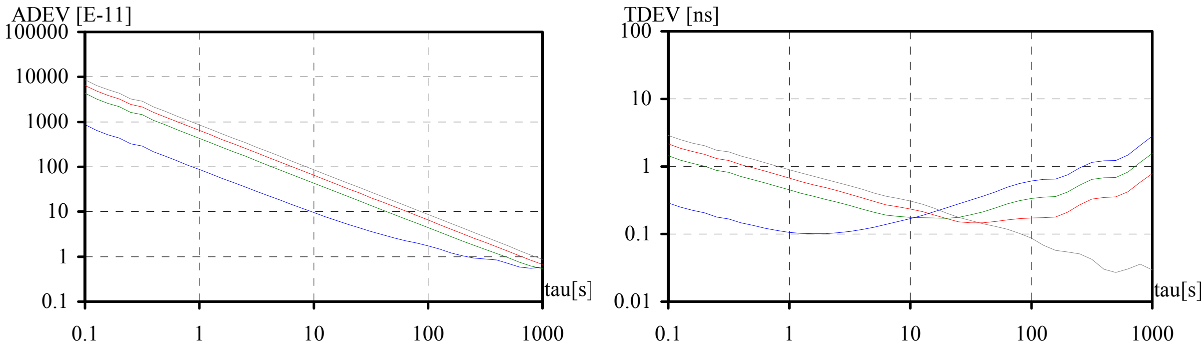

Fig. 6. ADEV and TDEV computed for the measurement instant i=120 000 samples (4000 s)

CONCLUSIONS

The results of the experimental tests have proved the ability of the real-time joint computation of Allan deviation and time deviation performed for time error samples obtained from several measurement channels. The multi-channel computation of the parameters can be jointly performed simultaneously for numerous series and wide range of observation intervals as well as rather short sampling interval. Multi-channel real-time analysis of timing signals can be very useful in the service and maintenance of the telecommunication networks.

ACKNOWLEDGMENT

This work was supported by the Ministry of Science and Higher Education in the frame of the project number N N517 1645 33.

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